CALCULATION OF POSITIONAL UNCERTAINTY FOR CADASTRAL SURVEYS

<u>Background</u>

At a recent forum held to discuss proposed changes to the Tasmanian Survey Directions, it was requested the Office of the Surveyor General provide information to the profession about the calculation of positional uncertainty.

The Tasmanian Survey Directions require at least two corners to be coordinated on the MGA. Additional corners must be coordinated so that no coordinated corners are more than 1 km apart, measured along the boundary. The Directions also specify *the maximum allowable positional error* allowed, however this term is shortly to be replaced by the term *positional uncertainty.*

This is in accordance with a new standard for expressing the accuracy of coordinates in Australia adopted by the Intergovernmental Committee on Surveying and Mapping (ICSM). In the ICSM Standards and Specifications for Control Surveys (SP1) Positional and Local Uncertainty are defined as follows:

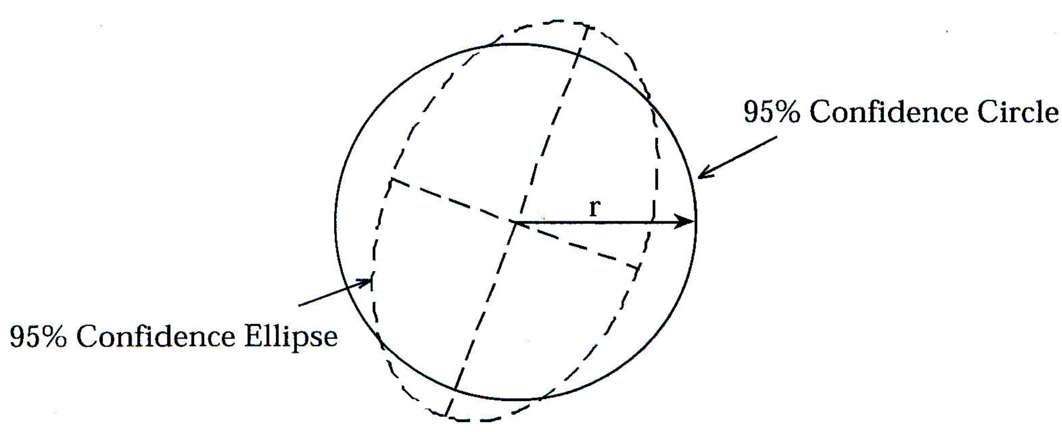
> *Positional Uncertainty is the uncertainty of the coordinates of a point, in metres, at the 95% confidence level, with respect to the defined reference frame. This value is the total uncertainty propagated from the datum, which in Australia is realised by the Australian Fiducial Network.*
>
> *Local Uncertainty is the average measure, in metres at the 95% confidence level, of the relative uncertainty of the coordinates, of a point, with respect to adjacent points in the defined frame.*

The Directions specify that the maximum positional uncertainty allowed depends on the distance from a permanent mark classified 2nd order or better:

Proximity of 2nd Order Mk	Max Positional Uncertainty
< 1 km	0.1 m (Urban areas)
1-5 km	0.3 m (Semi rural)
All other surveys	0.5 m (Rural)

The Directions require land surveyors to document the positional uncertainty of the coordinates they provide. It is important that realistic values are stated as it enables them to be used by other surveyors as a basis for coordinating adjoining or nearby surveys. Simply stating the positional uncertainty as the maximum allowed results in duplication of effort as subsequent surveyors will need to make unnecessary connections to permanent marks to coordinate their surveys.

Positional and local uncertainty can be determined for a wide range of spatial data using many different methods, including previous experience or manufacturers accuracy specifications. For geodetic surveying they are computed from appropriately scaled error ellipses. They are represented as a circle centered on the estimated horizontal position of a point as illustrated below. SP1 provides a formula for calculating the radius of this circle.



There are a number of computer programs, widely used for cadastral surveying that can be used to compute 95% confidence error ellipses. For example: Terramodel, Geocomp, Geocivil, Civilcad and Civil 3D. An Excel work sheet written by Bernie Williams as a 'front end' for the freely available program HAVOC computes both the error ellipses and the required confidence circles.

For the computation of realistic positional uncertainty values surveyors, need to be aware of the accuracy of the equipment they are using, the accuracy of the permanent marks they are using for an origin, and the way errors accumulate as they work. Whereas the computer programs mentioned above will compute rigorous values, the Survey Directions only require a realistic estimate.

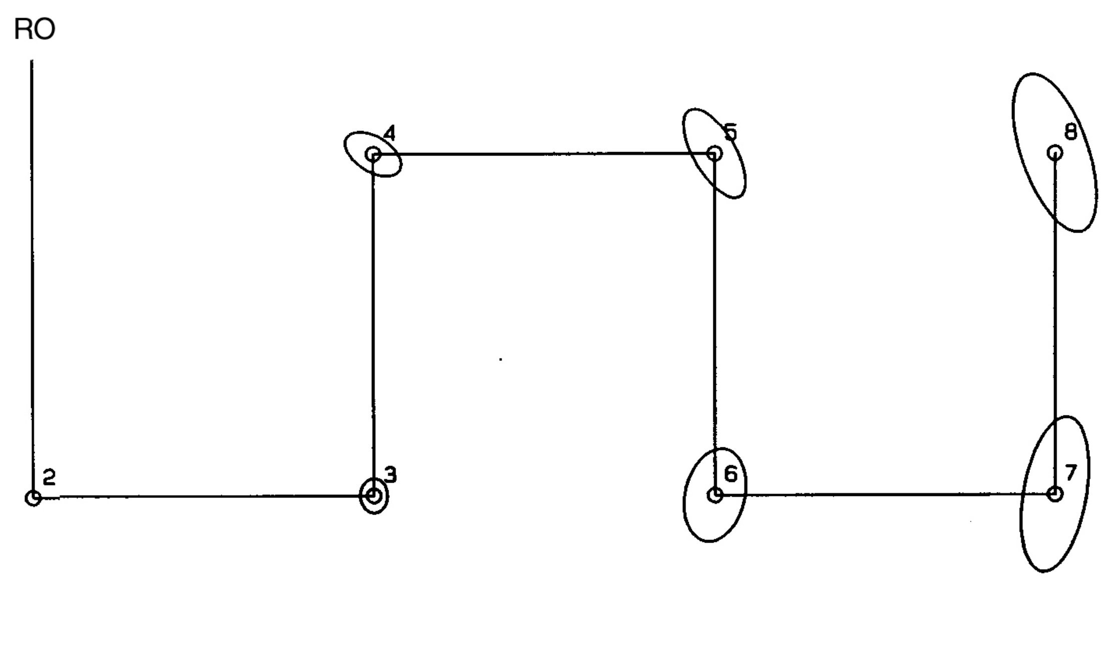
For GPS surveying this computation is relatively simple. Because there are usually very few instrument set-ups, centering errors can be ignored, and a realistic positional uncertainty (PU) computed from the accuracy specifications of the equipment or the rms value of the derived coordinates, both of which will be stated at a 68% confidence level. For example, a typical accuracy specification for RTK is 1 cm + 2ppm. For a 5 km baseline:

- The standard deviation of the coordinates will be 0.02 m (0.01+ 2ppm over 5 km)
- Because the coordinates are a 2 dimensional quantity, multiply by 2.45 to obtain a 95% confidence level (= 0.049)
- If the PU of the coordinate origin is 0.04, the PU of the derived coordinates will be $\sqrt{(0.04^2 + 0.049^{2)}} = 0.063$

When traversing with a total station the computation is not so simple as there are more sources of error to be allowed for. Typical standard deviations that can be assumed are:

- 5 mm + 3 ppm for distances
- 4" for angles
- 2 mm for forced centred plumbing
- 5 mm for prism pole plumbing

Accumulation of local uncertainty along a theoretical open ended traverse with 300 m legs carried out to transfer control to a cadastral survey. Origin is Stn 2



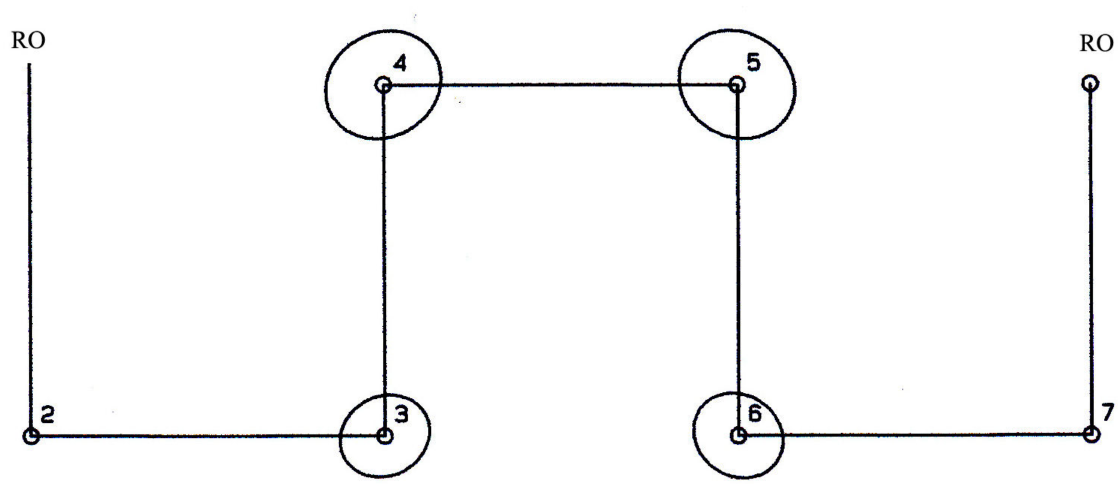
Accumulation of Local and Positional Uncertainty Along an Unclosed Traverse

Station	Dist from Stn 2	95% Confidence Point Ellipse		Uncertainty	
		Semi Maj.	Semi Min.	Local	Positional
2					0.040
3	300	0.030	0.014	0.025	0.047
4	600	0.054	0.026	0.045	0.060
5	900	0.092	0.030	0.075	0.085
6	1200	0.086	0.053	0.073	0.084
7	1500	0.154	0.051	0.125	0.131

From these results it is apparent that:

- The maximum length of an unclosed traverse that can be used to transfer control to a survey is approximately 700 m
- The radiuis of the uncertainty circle is approximately 83% of the length of the semi major axis of the 95% point ellipse.

Accumulation of Local and Positional Uncertainty Along a Constrained Traverse (Constrained at Stns 2 & 7)



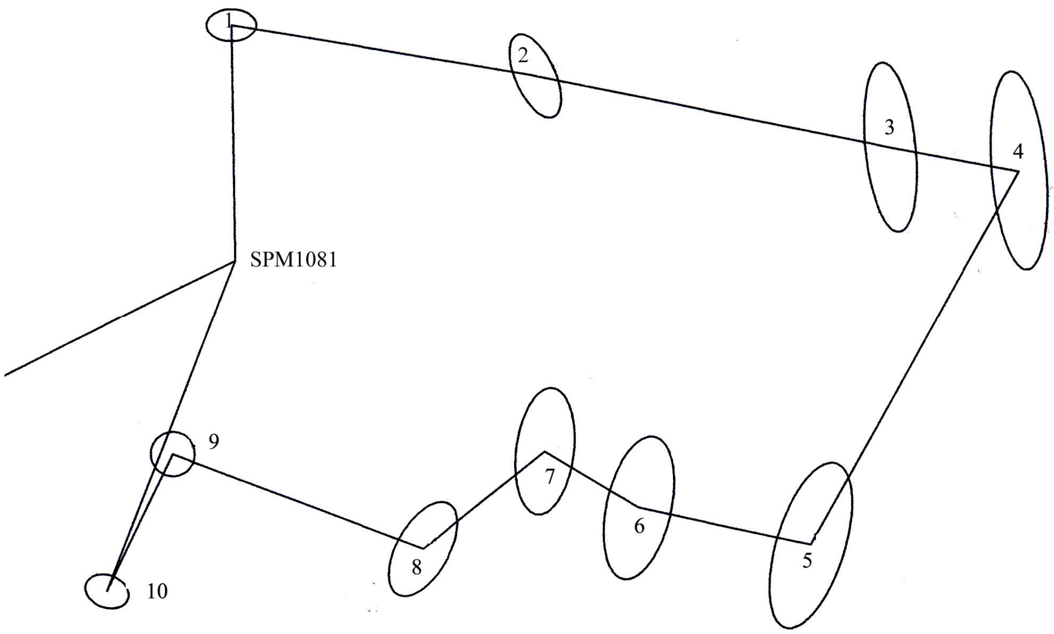
Total length of traverse 1500 m

Local uncertainty at 4 & 5 = 0.017

Positional uncertainty at 4 & 5 = 0.043

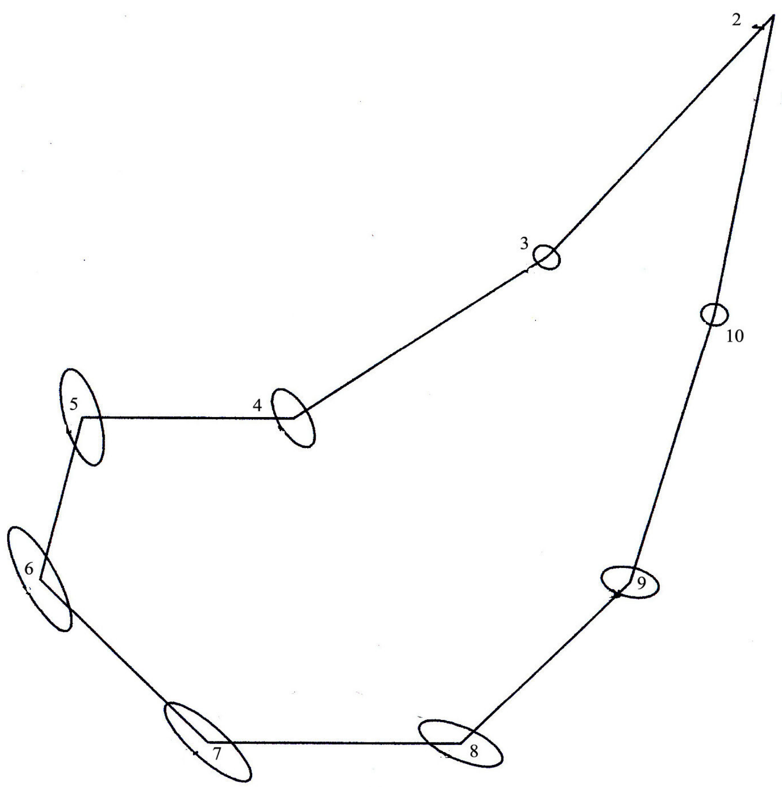
The size and shapes of the error ellipses illustrates how a traverse is considerably strengthened if it is constrained at both ends.

Accumulation of Positional Uncertainty around Traverses



Total length of traverse 608 m, average length of line 55 m

Positional uncertainty at 4 = $\sqrt{(0.04^2 + 0.038^2)}$
= 0.055



Total length of traverse 2800 m, average length of line 310 m

Positional uncertainty at 6 = $\sqrt{(0.063^2 + 0.066^2)}$
= 0.091

CONCLUSION

1. Because the shape of a traverse has such a dramatic effect on the propagation of coordinate uncertainty it is not possible to recommend a “rule of thumb” for calculating positional uncertainty (ie it increases xx mm per 100 m of traverse). The task is best left to a computer program.

2. Further examples of the calculation of positional error for different traverse configurations may be of assistance to surveyors for non-complex situations. These will be provided if required.

3. The radius of the uncertainty circle is approximately 83% of the length of the semi-major axis of the 95% point ellipse. This should be of interest to surveyors who’s software does not calculate confidence circles from ellipses.

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